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TIME STEP DEPENDENCE OF CREEP CALCULATIONS*

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SUMMARY

Creep calculations in support of the Strategic Petroleum Reserves require the use of an incremental solution with respect to time. The creep algorithms use a time step supplied by the user. Larger time steps may give erroneous results even though the computer solution is stable. This report presents the theory behind such problems and some examples showing several different solutions obtained where all circumstances were identical except the size of the time step.

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1. Introduction

Creep stability calculations are being done on leached salt caverns for storage of oil under the strategic petroleum reserves project. Analysis is also continuing on the Waste Isolation Pilot Plant near Carlsbad, New Mexico. Roth of these projects require accurate creep displacement analysis of openings in salt formations. Excessive creep displacement may result in either gradual closure or sudden failure of openings in the surrounding salt.

Recent code qualification exercises for ADINA78 [1] have been concentrated on simulation of triaxial creep tests performed by Wolfgang Wawersik (5532)[2]. During that exercise it was observed that some time steps, though resulting in stable computer solutions, gave erroneous results. The reasons for this have been investigated and are presented in this report.

The algorithms for obtaining creep strain along with displacements and stresses are similar in most of the finite element programs with creep capability. Even though the time step difficulties discussed here occurred in ADINA78, other programs will exhibit the same characteristics.

2. Theory

Among the nonlinear material models available in ADINA78 is the thermo-elastic-plastic and creep model which allows for creep strain based on one of three constitutive equations[3].

Calculation of creep strain and the resulting stresses is done by integrating the strain rate at a given time to give a strain increment which can be used to obtain a stress increment.

The incremental stress vector is calculated from

$$\Delta\underset{\sim}{\sigma} = \underset{\approx}{C}^{E}(\Delta\underset{\sim}{e} - \Delta\underset{\sim}{e}^{P} - \Delta\underset{\sim}{e}^{C} - \Delta\underset{\sim}{e}^{TH}) \tag{1}$$

where $\underset{\approx}{C}^{E}$ is the elastic stress-strain matrix and $\Delta\underset{\sim}{e}$, Δe^{P}, $\Delta\underset{\sim}{e}^{C}$, $\Delta\underset{\sim}{e}^{TH}$ are the total, plastic, creep and thermal incremental strain vectors. The creep model currently does not consider plastic or thermal strains so $\Delta\underset{\sim}{e}^{P}$ and $\Delta\underset{\sim}{e}^{TH}$ are equal to zero. Equation (1) then becomes

$$\Delta\underset{\sim}{\sigma} = \underset{\approx}{C}^{E} (\Delta\underset{\sim}{e} - \Delta\underset{\sim}{e}^{C}) \tag{2}$$

The incremental creep strain is determined by multiplication of a strain hardening parameter and the deviatoric stress as

$$\Delta\underset{\sim}{e}^{C}_{ij} = K\underset{\sim}{S} \tag{3}$$

where K is the strain hardening parameter and $\underset{\sim}{S}$ is the deviatoric stress. To solve equation (3) for the incremental creep strain, the strain hardening parameter K must be calculated. This is done in the following manner.

The current effective stress and.current effective strain are given in equations (4) and (5) respectively.

$$\bar{\sigma} = \left[\frac{3}{2} S_{ij} S_{ij}\right]^{\frac{1}{2}} \tag{4}$$

$$\bar{e}^{C} = \left[\frac{2}{3} e_{ij}^{C} e_{ij}^{C}\right]^{\frac{1}{2}} \tag{5}$$

The creep strain constitutive equation has the following form

$$\bar{e}^C = f(\bar{\sigma})(1 - \exp(-r(\bar{\sigma})t) + g(\bar{\sigma})t \qquad (6)$$

where f, r and g are functions of effective stress determined in the laboratory. The effective strain rate is given by the time derivative of equation (6)

$$\dot{\bar{e}}^C = f(\bar{\sigma})\, r(\bar{\sigma}) \exp(-r(\bar{\sigma})t) + g(\bar{\sigma}) \qquad (7)$$

Equations (4) and (5) are solved at each time step to give effective stress and effective creep strain. The two values are substituted into equation (6) and the time value satisfying equation (6) is found. This time value is a pseudo time that can be substituted into equation (7) to obtain the current creep strain rate. The strain hardening parameter is a function of the creep strain rate in the following way.

$$K = \frac{3}{2}\, \dot{\bar{e}}^C\, \Delta t/\bar{\sigma} \qquad (8)$$

Substituting (8) and (3) back into (2) we obtain equation (9)

$$\Delta \bar{\underset{\sim}{\sigma}} = \underset{\approx}{C}^E \left[\Delta \bar{\underset{\sim}{e}} - \frac{3}{2}\, \dot{\bar{e}}^C\, \Delta t/\bar{\sigma}\ \underset{\sim}{S}\right] \qquad (9)$$

Equation (9) can be integrated by an incremental numerical integration method where

$$\bar{\underset{\sim}{\sigma}}_n = \bar{\underset{\sim}{\sigma}}(n\Delta t) \qquad (10)$$

and

$$\bar{\underset{\sim}{\sigma}}_{n+1} = \bar{\underset{\sim}{\sigma}}_n + \Delta \bar{\underset{\sim}{\sigma}} \qquad (11)$$

The relationship between N and total time is given by

$$T = N\Delta t \tag{12}$$

or

$$At = T/N \tag{13}$$

where T is the final time, At is time step and N is the total number of increments. Equation (9) and (10) can be substituted into equation (11) to obtain

$$\underset{\sim}{a}_{n+1} = a_n + \underset{\approx}{C}^E \left[\Delta\underset{\sim}{\bar{e}}_n - \left(\frac{3}{2}\ \bar{e}^C\ \Delta t/\bar{\sigma}\right) \underset{\sim}{S}\right] \tag{14}$$

The incremental strain is related-to time and strain rate by

$$\Delta\underset{\approx}{e} = \Delta t\dot{\underset{\approx}{e}} \tag{15}$$

Equation (15) is substituted into equation (14) to give (16)

$$\underset{\sim}{a}_{n+1} = \underset{\sim}{\bar{\sigma}}_n + \underset{\approx}{C}^E\ At \left[\dot{\underset{\sim}{\bar{e}}}_n - \left(\frac{3}{2}\ \underset{\sim}{S}/\bar{\sigma}\right)\bar{e}^C\right] \tag{16}$$

The WIPP creep model proposed by Herrmann and Wawersik has the following form [4]

$$\bar{e}^C = AO\ (\bar{\sigma})^{A1}\ (1 - \exp(-A2(\bar{\sigma})^{A3}t) + A4(\bar{\sigma})^{A5}t \tag{17}$$

with the creep strain rate being the first derivative of equation (17)

$$\dot{\bar{e}}^C = -AO(^{t}\bar{a})^{A1}\ (-A2(^{t}\bar{\sigma})^{A3}\ \exp(-A2(^{t}\bar{\sigma})^{A3}t) + A4(^{t-}a)^{A5} \tag{18}$$

where AO to A5 are laboratory determined constants. Substituting (13) and (17) into (16) gives the numerical integral of the WIPP creep equation which is

$$\underset{\sim}{a}_{n+1} = \underset{\sim}{\bar{\sigma}}_n + \underset{\approx}{C}^E\ T/N \left[\dot{\underset{\sim}{e}}_n - \left(\frac{3}{2}\ \underset{\sim}{S}/\bar{\sigma}\right) \left(-A0(\bar{\sigma})^{A1} X \right.\right. \tag{19}$$

$$\left.\left. -A2(\bar{\sigma})^{A3}\ \exp\ -A2(\bar{\sigma})^{A3} t + A4(\bar{\sigma})^{A5}\right)\right]$$

where $\bar{\underset{\sim}{e}}$ is given by equation (18).

As N goes to infinity (At goes to zero) in equation (19), $\underset{\sim}{\bar{\sigma}}_{n+1}$ converges to $\underset{\sim}{\bar{\sigma}}_n$. But, as N goes to 1, $\underset{\sim}{\bar{\sigma}}_{n+1}$ becomes

$$\underset{\sim}{a}_n + \underset{\approx}{C}^E T \left[\dot{\bar{\underset{\sim}{e}}}_n - \left(\frac{3}{2}\ \underset{\sim}{S}/\bar{\sigma}(-A0(\bar{\sigma})^{A1}(-A2(\bar{\sigma})^{A3} X \right.\right. \tag{20}$$

$$\exp(-A2(\bar{\sigma})^{A3} t) + A4(\bar{\sigma})^{A5}$$

A value of N equal to 1. would give good results if equation (20) were linear. However, the exponential nature of equation (20) results in a large error when one time step is used because a linear approximation is being made of an exponential equation. To accurately integrate an exponential equation it must be broken into a number of small linear increments.

3. Examples

A triaxial creep test simulation using ADINA78 was completed on the finite element mesh shown in figure 1. This mesh models one quarter of the triaxial test specimen with symmetry boundary conditions used on the lower and left sides of the mesh. The analysis was axisymmetric. The nodes across the top were locked together vertically to simulate a rigid steel platen. Friction at the salt/steel interface was simulated by placing concentrated horizontal loads at the top nodes. A vertical pressure of 5000 psi was placed across the top and a horizontal pressure of 2000 psi was placed on the right side.

At the beginning of each creep calculation the time step was automatically increased until a time step long enough for economy but short enough for computer stability was reached. It was'discovered that time steps just short enough for computer stability resulted in erroneous calculations. Figure 2 shows the vertical strain versus time for three different time increment histories. The three curves result from analyses where no friction forces were applied to the top nodes. The time increment histories are given on the figure. The heavy line represents the constitutive equation with stress set to a constant 3000 psi. Figure 3 shows the vertical strain versus time when the salt/steel friction is set to .06. The error in this case is not as large but it still exists.

To further illustrate the effect of time step, the stress integration equations available in ADINA78 are examined in the following manner:

$$a_{n+1} = a_n + E\Delta t \left[\dot{e} - \dot{e}_c\right] \qquad (21)$$

where

E = Young's Modulus

σ = stress

At = time step

$\dot{e}$ = total strain

$\dot{e}_c$ = total creep strain

Generally, secondary creep dominates creep displacement so a common approach is to consider only secondary creep. Equation (18) is then reaouced to

$$\bar{e}_C = A4(\bar{\sigma})^{A5} \qquad (22)$$

Substituting (22) into (21) gives

$$\bar{\sigma}_{n+1} = a_n + E\,A\,t\left[\dot{\bar{e}} - A4(\bar{\sigma}_n)^{A5}\right] \qquad (23)$$

where

$E = 2 \times 10^6$ psi

$A4 = 1.6667 \times 10^{-20}$

$A5 = 4.9$

At = t/N

t = 20 days

Note: A4 and A5 are laboratory determined constants for units in psi and days;

When $\dot{\bar{e}} = 0$, equation (23) will model stress relaxation following a finite instantaneous strain. The approach taken is to eliminate $\dot{\bar{e}}$ from equation (23) to make simple integration possible. In figure 4, each point on the curve represents stress at twenty days with the value of stress being arrived at by integration of equation (23) over N increments.

Primary and secondary creep can be analyzed the same way by substituting all of equation (16) into equation (21) giving

$$a_{n+1} = a_n + E\Delta t\left[\dot{\bar{e}} + A0\,\bar{\sigma}_n^{A1} A2 0_n^{A3} \exp(A2\bar{\sigma}_n^{A3} t) - A4\bar{\sigma}_n^{A5}\right] \qquad (24)$$

where

$E = 2 \times 10^{6}$ psi

$AO = 5.97 \times 10^{-11}$

$Al = 2.38$

$A2 = -1.163 \times 10^{-9}$

$A3 = 2.52$

$A4 = 1.6667 \times 10^{-20}$

$A5 = 4.9$

$t = t/N$

$t = 20$ days

Again the assumption is made that $e = 0$ as in a stress relaxation problem. The results of integrating equation 24 are shown in figure 5.

The important item in figures 4 and 5 is the distinct elbow on the curve. Below this elbow the number of time steps is not large enough to correctly approximate the exponential function. Above the elbow a very flat line exists where essentially the same value of stress is obtained no matter how many increments are used. For creep calculations, the number of increments used should be just beyond the elbow for accuracy but not too far along the flat portion for economy. It is also interesting to note that errors due to large time steps will result in smaller than actual displacements and stresses. This is obvious in figures 2 and 3 where the erroneous strain curves were below the correct curve. It is also obvious in figures 4 and 5 where the erroneous stresses to the right of the elbow are below the correct stress. This causes concern because incorrect displacements due to time step problems will always be non-conservative from a structural failure point of view.

4. Conclusions

Rock salt creep analysis for structural design of salt caverns requires an incremental procedure to determine displacement and stresses. The selected length of the incremental time step is important for both computational economy and accuracy. A time step that is too long will result in error since the exponential creep function is being approximated over a long time step by a linear function. Time steps that are excessively short will not increase accuracy and will be uneconomical. There appears to be a critical point in time step length below which the solution is within the required accuracy and above which the solution rapidly deteriorates with increasing time step length.

The error resulting from a time step that is too long appears to be nonconservative in that displacements and stresses are smaller than actually occur.

5. References

[1] Bathe, Klaus-Jurgen, "ADINA -- A Finite Element Program for Automatic Dynamic Incremental Nonlinear Analysis," Report 82448-1, MIT, Cambridge, MA, December 1978.

[2] "Creep Curves and Fitting Parameters for Southeastern New Mexico Bedded Salt," SAND80-0087, Sandia Laboratories, Albuquerque, NM, March 1980.

[3] Bathe, Klaus-Jurgen, "Static and Dynamic Geometric and Material Nonlinear Analysis Using ADINA." Report 82448-2, MIT, Cambridge, MA, May 1976.

[4] Wawersik, W. R., "In-situ Structural Heater Experiment in Support of WIPP," Memo to J. D. Plimpton, December 20, 1979.

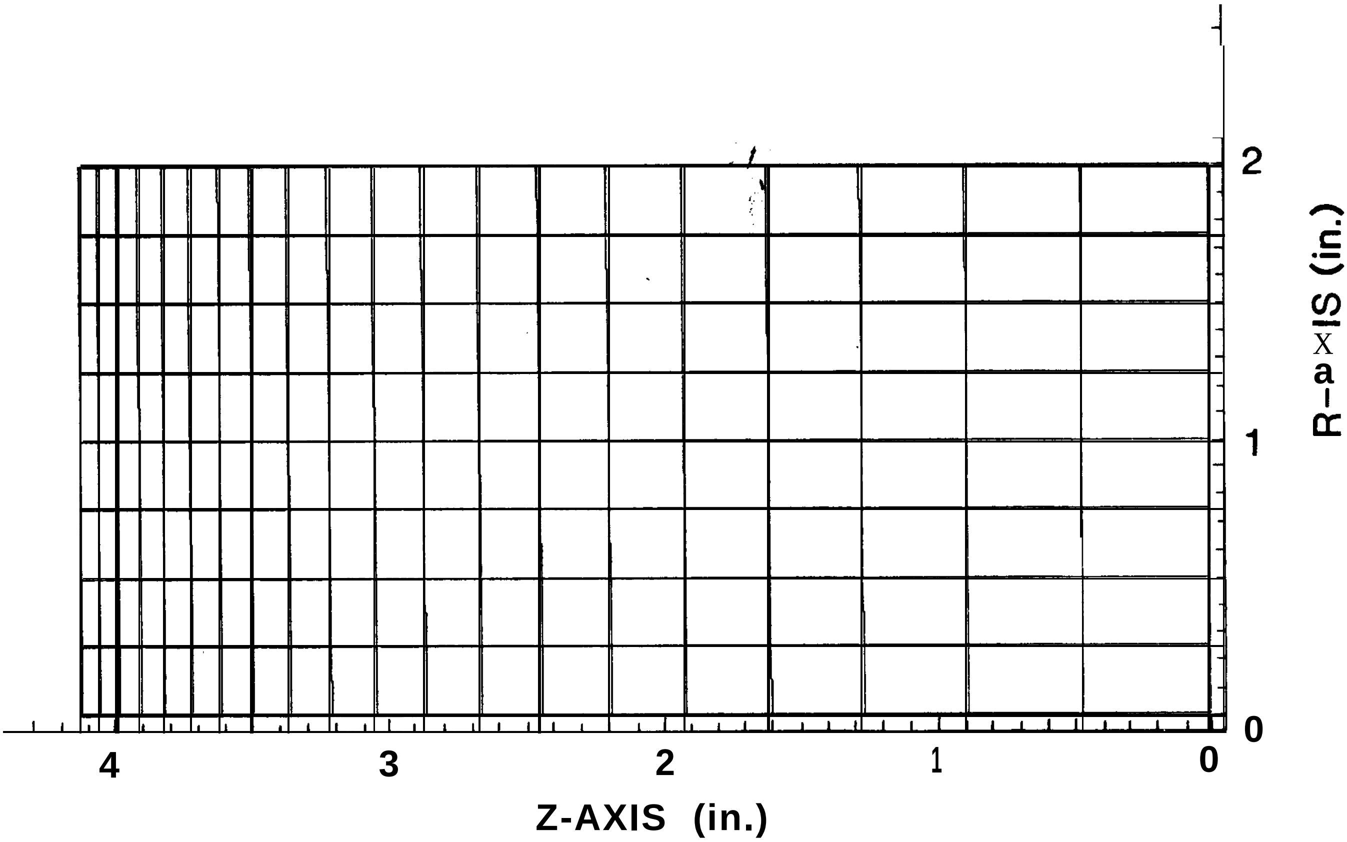
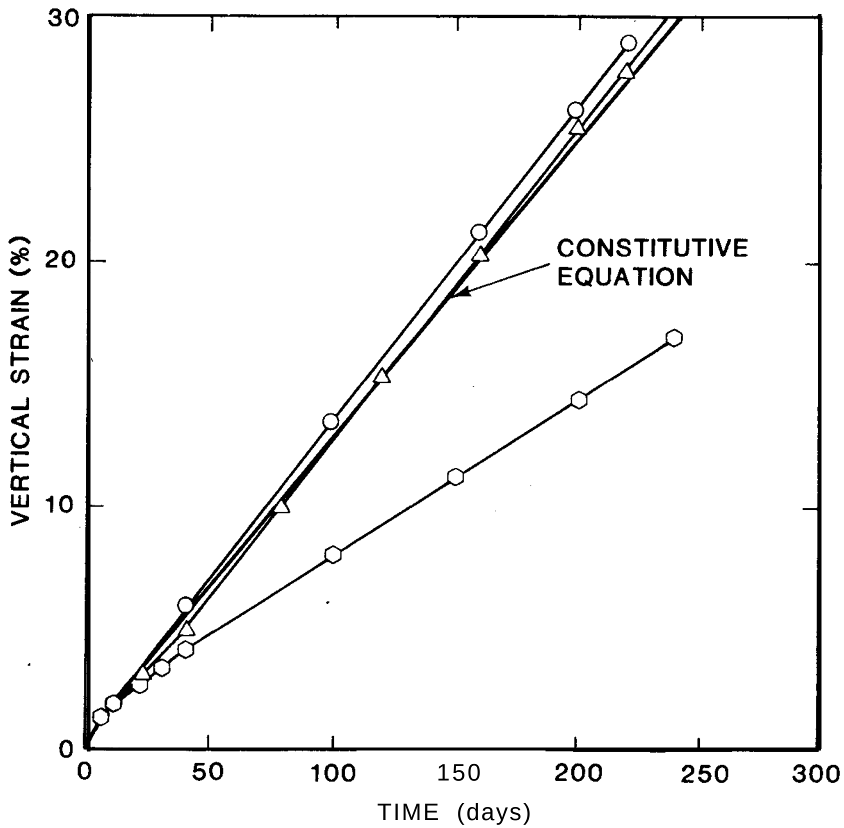


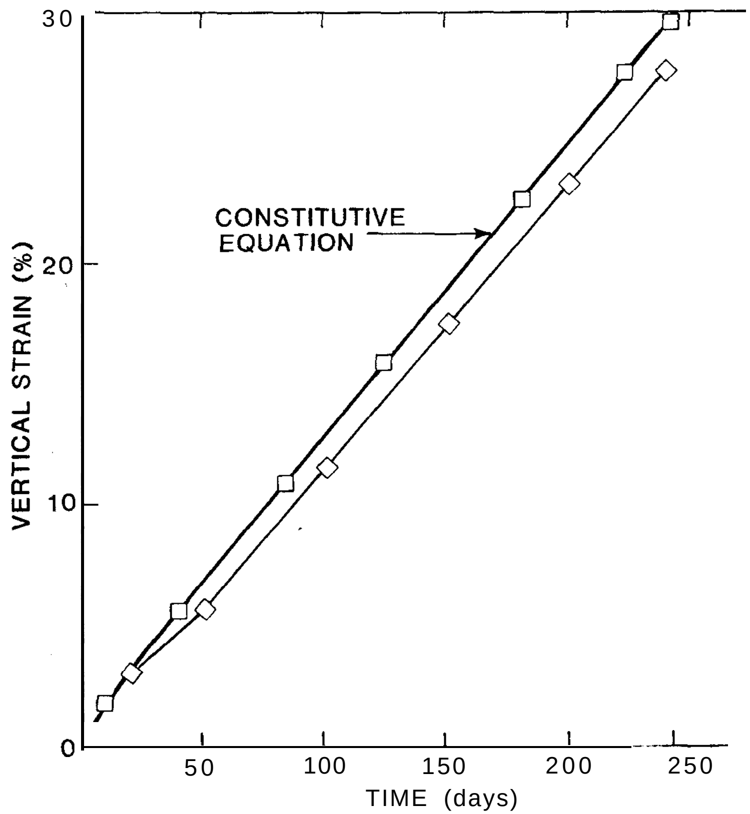
Figure 1
Finite Element Mesh for Triaxial Creep Test



SYMBOL	INCREMENT (n)	TIME PERIOD (days)	TIME INCREMENT (days)
0	O-90	O-58.43	$0.001(1.1)^n$
	90-124	58.43-240	5.3 1
△	o-77	0- 16.92	$0.001(1.1)^n$
	77-222	16.92-240	1.53
0	O-65	O-5.383	$0.001(1.1)^n$
	65-534	5.383-240	0.50

ALL CALCULATIONS WERE DONE WITH ZERO FRICTION AT THE SALT/STEEL INTERFACE.

Figure 2
Time Step Dependence of Vertical Strain



SYMBOL	INCREMENT (n)	TIME PERIOD (days)	TIME INCREMENT (days)
0	o-77 77-222	O-16.92 16.92-240	$0.001(1.1)^n$ 1.53
□	O-65 65-534	O-5.38 5.38-240	$0.001(1.1)^n$ 0.50

ALL CALCULATIONS WERE DONE WITH A COEFFICIENT OF FRICTION EQUAL TO 0.06 AT THE SALT/STEEL INTERFACE.

Figure 3

Time Step Dependence of Vertical Strain

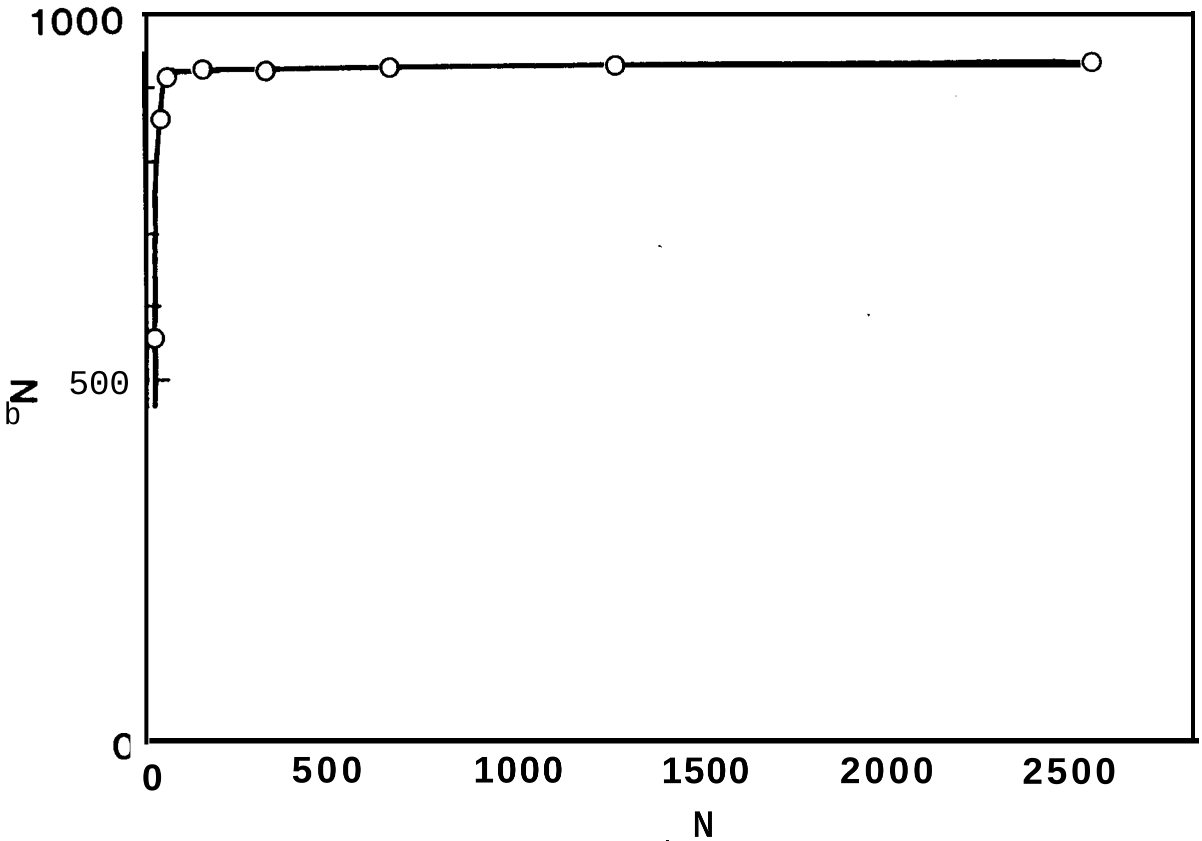


Figure 4

σ_N vs. N For Secondary Creep Only

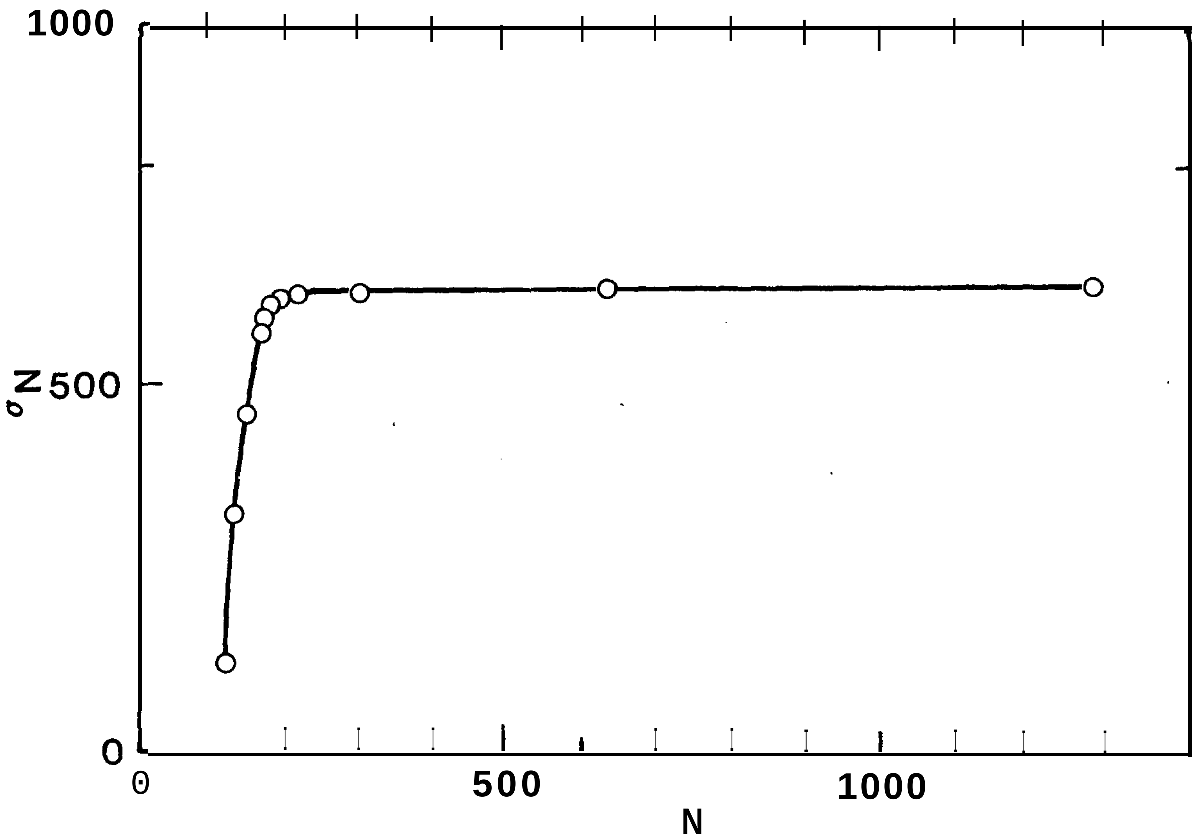


Figure 5

σ_N vs. N for Primary and Secondary Creep

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